New Keynesian Model with Robust Agents

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Introduction

- Introduce a robust household fearing model misspecification into New Keynesian (NK) model.
- Compared with standard NK model, the belief distortion plays a key role in the equilibrium
  - SDF, Euler equations, price setting rule

Households with Robustness

Households maximize

$$U_t = \max_{C_t, N_t} \min_{\substack{Z_{t+1}>0 \\ \mathbb{E}_t[Z_{t+1}]=1}} u\left(C_t, N_t\right) + \beta \mathbb{E}_t\left[Z_{t+1} U_{t+1}\right] + \frac{\beta}{\theta_t} \mathbb{E}_t\left[Z_{t+1} \log Z_{t+1}\right]$$

subject to

$$P_t C_t + \frac{B_{t+1}}{R_{t+1}} \leqslant D_t + W_t N_t + B_t$$

- optimal belief distortion

$$\frac{d\widetilde{P}}{dP}\Big|_{\mathcal{F}_{t+1}} = Z_{t+1} = \frac{\exp\left(-\theta_t U_{t+1}\right)}{\mathbb{E}_t\left[\exp\left(-\theta_t U_{t+1}\right)\right]}$$

- recursive utility

$$U_t = \max_{C_t, N_t} u\left(C_t, N_t\right) - \frac{\beta}{\theta_t} \log \mathbb{E}_t\left[\exp\left(-\theta_t U_{t+1}\right)\right] \tag{1}$$

Example

Consider a special period utility

$$u(C_t, N_t) = \frac{C_t^{1-\sigma} - 1}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi} \tag{2}$$

Combining F.O.Cs and envelop conditions,

- Intertemporal condition: With real MRS $M_{t+1} = \beta \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma}$,

$$1 = \widetilde{\mathbb{E}}_t \left[M_{t+1} \frac{P_t}{P_{t+1}} \right] R_{t+1} \tag{3}$$

- Intratemporal condition:

$$\frac{W_t}{P_t} = C_t^{\sigma} N_t^{\varphi} \tag{4}$$

Final Goods

- Final goods Y_t with price P_t with production technology

$$Y_t = \left[\int_0^1 Y_t(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}}$$

- Final good producers solve the profit maximization problem, yielding demand schedule

$$Y_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\varepsilon} Y_t$$

Intermediate goods

- Intermediate goods $Y_t(i)$ produced via technology

$$Y_t(i) = A_t N_t(i)^{1-\alpha}$$

where aggregate TFP process $a_t \equiv \log(A_t)$

$$a_t = (1-\rho_a)a^* + \rho_a a_{t-1} + w_t^a, \quad w_t^a \sim N(0, \sigma_a^2)$$

- Firm maximizes profit with quadratic adjustment cost (Rotemberg 1982)

$$V_t^{(i)} = \max_{P_t(i)} \frac{P_t(i)}{P_t} Y_t(i) - mc_t Y_t(i) - \frac{\phi_R}{2}\left(\frac{P_t(i)}{P_{t-1}(i)} - 1\right)^2 Y_t + \widetilde{\mathbb{E}}_t\left[M_t V_{t+1}^{(i)}\right]$$

$$\text{s.t. } Y_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon} Y_t$$

Optimal price setting rule

Under the symmetric equilibrium, the price should be set as

$$(1-\varepsilon)+\varepsilon mc_t = \phi_R\left(\Pi_t - 1\right)\Pi_t - \phi_R \widetilde{\mathbb{E}}_t\left[M_{t+1}\left(\Pi_{t+1}-1\right)\frac{Y_{t+1}}{Y_t}\Pi_{t+1}\right]$$

where $\Pi_t = \frac{P_t}{P_{t-1}}$.

- Under flexible price($\phi_R = 0$),

$$mc_t^n = \frac{MC_t}{P_t^n} = \frac{\varepsilon - 1}{\varepsilon}$$

New Keynesian Phillips Curve

Log-linearizing optimal price setting rule around the steady state, new Keynesian Phillips Curve is

$$\hat{\pi}_t = \gamma_1 \hat{mc}_t + \gamma_2 \widetilde{\mathbb{E}}_t \hat{\pi}_{t+1} \tag{5}$$

where $\gamma_1 = \frac{\varepsilon-1}{\phi_R}$ and $\gamma_2 = \beta \Delta Y_{ss}^{1-\sigma}$, if one defines $\Delta Y_t = \frac{Y_t}{Y_{t-1}}$.

- With production function $Y_t = A_t N_t^{1-\alpha}$,

$$\hat{\pi}_t = k\left(y_t - y_t^n\right) + \beta \widetilde{\mathbb{E}}_t \hat{\pi}_{t+1}$$

  where $k = \frac{\varepsilon-1}{\phi_R}\left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right)$, y^n is log of natural output.

- Natural output

$$y_t^n = \underbrace{\frac{1+\varphi}{\sigma(1-\alpha)+\varphi+\alpha}}_{\Gamma_a} a_t + \underbrace{\frac{(1-\alpha)\ln(1-\alpha) - (1-\alpha)\ln\frac{\varepsilon}{\varepsilon-1}}{\sigma(1-\alpha)+\varphi+\alpha}}_{\Lambda}$$

IS Curve

Log-linearizing intertemporal condition (3) around the steady state, IS curve is

$$\widetilde{\mathbb{E}}_t \tilde{y}_t = \widetilde{\mathbb{E}}_t \tilde{y}_{t+1} - \frac{1}{\sigma}\left(i_t - \widetilde{\mathbb{E}}_t \hat{\pi}_{t+1} - r_t^n\right) \tag{6}$$

where $\tilde{y}_t = y_t - y_t^n$ is the output gap and i_t is the short-term nominal interest rate.

- With $i_t = \ln R_{t+1}$

$$i_t = \rho + \widetilde{\mathbb{E}}_t \hat{\pi}_{t+1} + \sigma \widetilde{\mathbb{E}}_t \left[y_{t+1} - y_t\right]$$

  where $\rho = -\ln \beta$

- Under the flexible price, the natural rate

$$r_t^n = \rho + \sigma \widetilde{\mathbb{E}}_t \left[y_{t+1}^n - y_t^n\right]$$

Equilibrium

Government sets the nominal interest rate based on Taylor rule,

$$i_t = i^* + \phi_\pi \hat{\pi}_t + \phi_y \tilde{y}_t \tag{TR}$$

Along with IS and Philips Curve,

$$\hat{\pi}_t = k\tilde{y}_t + \beta \widetilde{\mathbb{E}}_t \hat{\pi}_{t+1} \tag{PC}$$

$$\widetilde{\mathbb{E}}_t \tilde{y}_t = \widetilde{\mathbb{E}}_t \tilde{y}_{t+1} - \frac{1}{\sigma}\left(i_t - \widetilde{\mathbb{E}}_t \hat{\pi}_{t+1} - r_t^n\right) \tag{IS}$$

These define the equilibrium in this special example.

- only one shock in this model, from TFP a_t
- solve $(i_t, \hat{\pi}_t, y_t)$ by three dynamics above and natural output/rate,

$$y_t^n = \Gamma_a a_t + \Lambda, \quad r_t^n = \rho + \sigma \widetilde{\mathbb{E}}_t \left[y_{t+1}^n - y_t^n\right]$$

Bond Price

The nominal marginal rate of substitutions is

$$\mathcal{M}_t^{\$} \equiv M_t/\Pi_t = \beta \left(\frac{C_t}{C_{t-1}} \right)^{-\sigma} \frac{P_{t-1}}{P_t}$$

With belief distortions Z_t, the nominal SDF is

$$\mathcal{S}_t^{\$} \equiv \mathcal{M}_t^{\$} Z_t$$

The price of an n-period nominal bond $\mathcal{P}_t^{(n)\$}$ can be written recursively as

$$\mathcal{P}_t^{(n)\$} = \mathrm{E}_t \left[\mathcal{S}_{t+1}^{\$} \mathcal{P}_{t+1}^{(n-1)\$} \right]$$

with $\mathcal{P}_t^{(0)\$} = 1$ and $\mathcal{P}_t^{(1)\$} = 1/R_{t+1}$.

Approximating Belief Distortion

- Belief distortion 1^{st}-order approximation (Bhandari, Borovicka and Ho 2019), with $\theta_t \equiv \theta$

$$Z_{t+1} = \frac{\exp\left(-\theta U_{t+1}\right)}{\mathbb{E}_t\left[\exp\left(-\theta U_{t+1}\right)\right]} \approx \frac{\exp\left(-\theta U_x \psi_w w_{t+1}\right)}{\mathbb{E}_t\left[\exp\left(-\theta U_x \psi_w w_{t+1}\right)\right]}$$

- x is a in our setting with $a_t = \psi(a_{t-1}, w_t^a) = (1-\rho_a)\, a^* + \rho_a a_{t-1} + w_t^a$, then $\psi_w = 1$ and

$$U_a = \frac{\partial U_{t+1}}{\partial a_{t+1}} = \frac{\partial U_{t+1}}{\partial C_{t+1}} \cdot \frac{\partial C_{t+1}}{\partial A_{t+1}} \cdot \frac{\partial A_{t+1}}{\partial a_{t+1}} = e^{(1-\sigma)a_{t+1}} N_t^{(1-\sigma)(1-\alpha)}$$

- Under the new measure, the TFP shock can be approximated by

$$w_{t+1}^a \sim N\left(-\theta U_a \psi_w, I\right)$$

Impact of Negative Shock

- Negative shock $t \rightarrow a_t \downarrow \rightarrow a_{t+1} \downarrow$ due to positive autocorrelation
  $\rightarrow U_a \uparrow$ if $\sigma > 1$, as N_t is unchanged under equilibrium
  $\rightarrow$ expectation of shock w^a_{t+1} will be more negative
- If $\sigma = 1$ (i.e. the household has log utility on C_t), the expectation of future shock will be not effected by past shocks.
- For the natural rate r^n_t,

$$r^n_t = \rho + \frac{\sigma(1+\varphi)}{\sigma(1-\alpha)+\varphi+\alpha}\Big((1-\rho_a)(a^* - a_t) + \widetilde{\mathbb{E}}_t\left[w^a_{t+1}\right]\Big)$$

  a negative shock (similar to climate risk) will decrease the natural rate with $\sigma > 1$, which is also the real interest rate as inflation is 0 with flexible price

Adding Rare Disasters

- Approximate belief distortion with first or higher orders, like Bhandari, Borovicka and Ho(2019) does
- Solve the equilibrium and SDF with other numerical techniques
  - Adding robustness, household's utility becomes a special recursive utility (1)
  - Bidder and Smith(2012) solved belief distortion for the robust agent with the algorithm for recursive utility
  - Fernández-Villaverde and Levintal(2018) solved DSGE with recursive utility and rare disasters